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A Contour Map of the Moon

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by

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ABSTRACT

The lunar elevation data provided by Schrutka-Rechtenstamm supplemented by similar data near the limbs provided by Davidson and Brooks, were used to determine the equation of surface of the Moon as a sum of spherical harmonics including terms up to the eighth order. This equation was used to construct a contour map of the Moon. The same work was repeated for data provided by Baldwin which also were supplemented by the same data by Davidson and Brooks. The two maps have some basic similarities. After improving the equation of surface derived from Schrutka's data in a way that a homogeneous Moon with this equation of surface would also satisfy the conditions, $f = 0.633$ (Koziel, 1964), $\beta = 0.0006267$ (Jeffreys, 1959), $\gamma = 0.0002274$ (Jeffreys, 1961), we derive a third contour map which seems to be more realistic than the previous two.

I. INTRODUCTION

The first contour map of the Moon was constructed by Franz (1899) just before the turn of the century. He employed only 55 points, a number not too low when sufficiently accurate, or when expectations are not exceeding the possibilities. He worked with linear interpolation between the known points, a rather poor technique for the present and in this particular case. His results appear, however, very interesting for a number of reasons. First, similarities between his map with the map produced in 1963 by Baldwin and the map presented here on the basis of Baldwin's data are obvious and many. It shows, in conformity with the latter two maps, that part of Mare Tranquillitatis, Mare Serenitatis, Mare Imbrium, part of Mare Frigoris, and Oceanus Procellarum are below the mean lunar level. Second, the Terrae of the southern hemisphere are highlands. Finally, a careful study of this map leads easily to the conclusion that if the far side of the Moon is symmetrical to the near one, an assumption very probably true, then the lunar figure, as well as potential, acquire a non-negligible fourth order harmonic. The basic argument in this line is that if the Moon was a triaxial ellipsoid, then the contour lines around Oceanus Procellarum, Mare Imbrium and Mare Serenitatis of elevation -1.2 and -2.4 kilometers should not be followed in the direction toward the northeastern limb by a zero elevation line but by an even lower elevation. It appears that the central bulge of the Moon is followed by depressions at approximately half the way towards

the limbs which finally are succeeded by highlands only in the direction of, approximately, the north lunar pole and the east-west diameter. The southern hemisphere does not fit completely in this pattern and, as we shall see later, this is so for the contour map given by Baldwin. The reason is that the lunar figure possesses a third order zonal harmonic, i.e., a pear-shaped component, which results in an asymmetry between the northern and southern hemispheres. By an argument of analogy to the Earth's similar harmonic, Kaula, (1963), has deduced that this asymmetry results in a term

$$K_{30} \frac{GM'}{R} \left(\frac{r_0}{R}\right)^3 T_{30} , \tag{1}$$

in the gravity field of the Moon, where G is the Gaussian constant, r_o and M' the mean radius and mass of the Moon, R the selenocentric distance of the attracted body, and T_{30} the third order Legendre polynomial. Kaula has estimated that the order of magnitude of K_{30} is $\pm 9.3 \times 10^{-5}$. The present author, (1964), has given

$$K_{30} = - 8.63 \times 10^{-5} , \tag{2}$$

and the agreement, if not accidental, is satisfactory. The solid harmonic (1) is the result of the surface harmonic,

$$j_{30} \, r_0 \, T_{30} , \tag{3}$$

and it is shown (Goudas, 1964) that

$$j_{30} = \frac{3}{7} K_{30} . \tag{4}$$

The "pear-shape" term in the lunar figure partly conceals, therefore, the existence of an even larger fourth order harmonic, a fact difficult to ascertain without a harmonic analysis of hypsometric data. Once such an analysis is made, mere inspection of Franz's map verifies the fact. We shall see that this is also true for the maps by Baldwin and Schrutka-Rechtenstamm.

The map by Ritter (1934) was the next to appear. Measurements of elevations were made by him on the basis of the terminator technique which has not received a final qualification even today. Hopmann (1964) proposes to apply this technique somewhat differently and avoid the inaccuracies due to the vagueness of the curve of the terminator. Ritter's results seem to have large systematic errors due exactly to this reason, and his map does not seem to be accurate or useful.

Schrutka-Rechtenstamm and Hopmann (1958) published a map based on the 150 points measured initially by Franz (1901) and subsequently remeasured by Schrutka-Rechtenstamm (1958). The mean error in the latter measurement is about 1.23 km. and the bulge deduced from them is about 3 km. according to Schrutka, the assumption being that the Moon is a triaxial ellipsoid. Approximately the same size of bulge was deduced from the same data by this author (1963) but without this assumption. The current values of f and β of the Moon are decisively against this large ellipsoidal value of the bulge which is overestimated for reasons explained by this author (1964). Fortunately enough the map by Schrutka and Hopmann is not based on this assumption and in spite of the large mean error, it

represents a considerable improvement of the map by Franz. The optical impression one has from the telescopic view of the Moon has been completely disregarded by Franz and Schrutka and Hopmann in the compilation of their maps which, therefore, are the presentation of unbiased measurements. Thus, according to the map by Schrutka and Hopmann Mare Nubium, part of the Oceanus Procellarum, part of Mare Imbrium, part of Mare Tranquilitatis, Mare Vaporum and Sinae Medii and Aestuum are above the mean lunar level, in contradiction to the general expectation that Maria must in general be below mean level. As it will later be pointed out, the harmonic analysis of the figure of the Moon has shown that this result can very well be true. The frame of "standard coordinates" and the principal axes of inertia should coincide and a "balanced" contour map of the Moon must always satisfy this requirement. The possible inhomogeneity of the Moon makes it difficult to decide whether a map satisfies this requirement or not. In case the density depends only on the distance from the center or if the Moon is nearly homogeneous, the above condition is satisfied if the mass over the mean lunar level is centered around the Earth-Moon line or the $O\zeta$-axis. The same must be approximately true for the low lands or the depression areas. This argument can be reciprocated and used as a criterion in consideration of the density distribution of the Moon. For example, if it is assumed that the map of Schrutka and Hopmann is correct, it can be concluded that either the lunar density depends only on the distance from the center or that the Moon is approximately homogeneous. From this point of view the contour map by Franz seems

slightly off balance.

The contour map by Baldwin (1963) was based on more points (696) measured from five Lick plates. A harmonic analysis of the lunar figure based on these measurements has verified the conclusions of Baldwin in connection with the bulge. In addition, the equation of surface obtained from the same analysis made possible the construction of a contour map which is similar to the map by Baldwin but not in minute details. This, of course, is expected since we have employed harmonics up to the eighth order only and thus the details of the elevation data derived from plate measurements were smoothed out leaving the general underlying pattern of the contour map. Most of the conclusions drawn by Baldwin in connection with his contour map have in this way been independently verified. We shall discuss further this map later on. In the meantime it should be mentioned that the elevation data by Baldwin have received some strong criticism by Hopmann (1964) and the present author (1964) on two different bases. Hopmann has announced that the results of measurement of the same 18 formations made by Baldwin (1963), by Schrutka (1958-1964), and by the Army Map Service (1960) - (1964) are very poorly correlated. According to the same author the best measurements at present are those of the Army Map Service followed in the second place by the unpublished measurements by Schrutka, who has used eight Lick plates. Finally, in last place comes the data by Baldwin. It must be pointed out, however, that the non-existence of strong correlations between measurements of the same quantities cannot decide undisputably which measurements are the best. Nevertheless, the present author (1964) has concluded

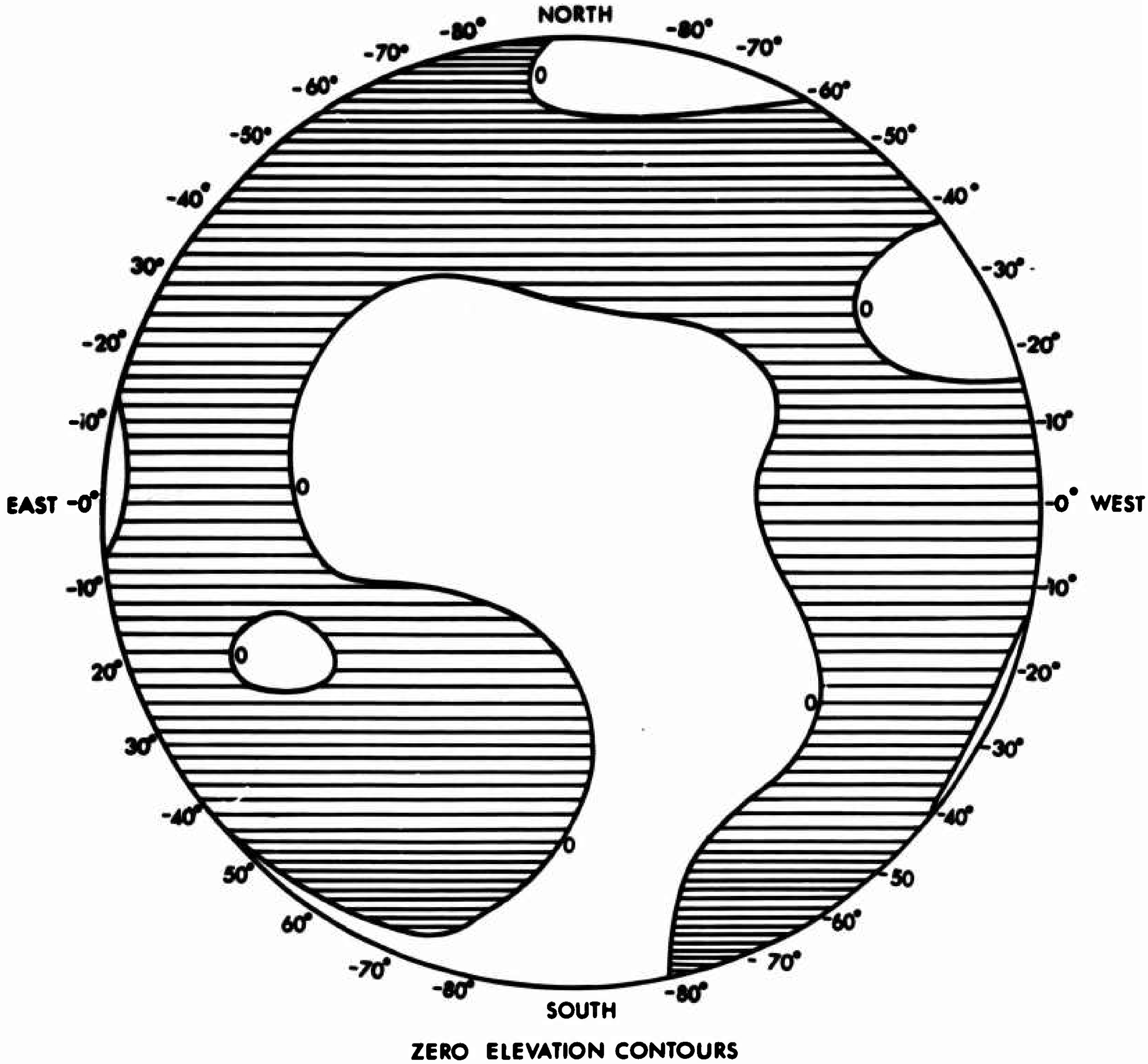
that the measurements by Schrutka (1958) are more accurate in comparison to the measurements by Baldwin (1963) and in this respect he is in agreement with the opinion of Hopmann. The basis of this comparison is much stronger and the conclusion definitive when established. A set of absolute high measurements, as a whole, must be compatible with the measurements of f, β and γ obtained from librations. On the assumption that the Moon is nearly homogeneous or that its density varies but only with the distance from its center and not with the direction, it is found that Schrutka's measurements are compatible with the correct values of f, β and γ. It has also been shown that Baldwin's measurements are leading to a negative value for the mechanical ellipticity f, and to unacceptable values for β and γ. The data derived by the Army Map Service and by Schrutka (1964) were not available to the author at the time of this publication and so he cannot agree or disagree with Hopmann in connection with the accuracy of these data. It is hoped that the results of their harmonic analysis will be published in the near future.

The contour map of the Army Map Service (1964) is based on 256 points evenly distributed over the visible face of the Moon. The average probable error in the elevations given is given to be about 858 meters which is about the same with the average errors in the data of Baldwin but definitely smaller than Schrutka's average error 1370 meters). Although it is impossible to assess the real value of this set of measurements and, therefore, of the corresponding

hypsometric map before a harmonic analysis is performed on them, we can say that this map has more points in common with the map by Schrutka and Hopmann than with the map by Baldwin. For example, Mare Imbrium is deep everywhere according to Baldwin, it is half below level (nowhere lower than 1 km.) and half above level (nowhere more than 1 km.) according to Schrutka and Hopmann, and finally it is almost everywhere above level with some parts reading the level of 3 km., according to the map of the Army Map Service. Another example is Mare Nubium which, according to the maps of Schrutka and Hopmann and the Army Map Service, is above level, and according to Baldwin's map, is below level.

II. A TECHNIQUE FOR CONSTRUCTION OF CONTOUR MAPS

As is pointed out by Hopmann (1964) and others, most of the existing measurements of absolute heights can be very accurate if treated as relative heights between consecutive lunar formation, but it is very doubtful whether they represent accurate relative heights for formations quite apart. The stereoscopic technique can be more trustworthy in this respect but this is not clear as yet. For this reason maps based on such measurements are bound to have more value as maps of relative heights with the merit that they show with sufficient accuracy the elevation relationships of nearby formations everywhere on the Moon. In other terms, they are locally and everywhere correct within the claimed accuracies. However, they do not give absolute contour lines or hypsometric curves with respect to the center of the Moon. It must be realized that lunar chartography



ZERO ELEVATION CONTOURS

FIGURE I

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is clearly divided in "<u>relative</u>" and "<u>absolute</u>" and the two are different in substance and in purpose. For instance the map by Baldwin can be regarded as a good example of a map of relative contours so long as we do not try to correlate distant formations. The contour map by Schrutka and Hopmann is more like, but not exactly, one of the second kind. Maps of absolute contours must satisfy additional necessary conditions which those of relative contours do not need to satisfy. This is exactly the basis on which improved absolute contour maps can be derived from data which nearly, but not exactly, satisfy these necessary conditions. If the data do not satisfy them even approximately, then they must be rejected. In this article it will be shown how this can be done in theory and how this was applied in the case of the data by Schrutka-Rechtenstamm.

At first we determine with the aid of elevation data the equation of surface of the Moon in the form of a sum of spherical harmonics. It is best if we include in this equation sufficient terms so as to make the mean squared error of the least squares fit smaller than the mean squared error of the observations. This may perhaps imply the inclusion of harmonics up to the 10th order or even higher. Second, it is necessary to make some assumption about the other side of the Moon. Two assumptions have been tested in connection with this. First, it was assumed that the lunar surface is symmetrical with respect to the plane $O\xi\eta$ of the standard frame of reference and, second, that it is symmetrical with respect to the origin of the standard frame. The values of J_{20} and J_{22} obtained from a least square fit of the data and based separately on each one of

these two assumptions were compared afterwards with the well-known values of J_{20} and J_{22} of the Moon as these are computed with the aid of the current values of f, β and γ. As a result of this comparison the second assumption was rejected because it led to values for J_{20} and J_{22} for worse than those found by means of the first assumption about the far lunar side. In detail it was found that for the elevation data by Schrutka and for the first assumption the values of J_{20} and J_{22} were:

$$J_{20} = -0.10 \quad , \qquad J_{22} = 0.31, \tag{5}$$

whereas for the same data and for the second assumption they were:

$$J_{20} = 0.23 \quad , \qquad J_{22} = 0.46 \; . \tag{6}$$

The data by Baldwin for the same two assumptions gave, respectively,

$$J_{20} = 0.94 \quad , \qquad J_{22} = -0.030, \tag{7}$$

and,

$$J_{20} = 0.98 \quad , \qquad J_{22} = -0.027 \; . \tag{8}$$

The correct values (Goudas, 1964) are:

$$J_{20} = -0.59 \quad , \qquad J_{22} = 0.067, \tag{9}$$

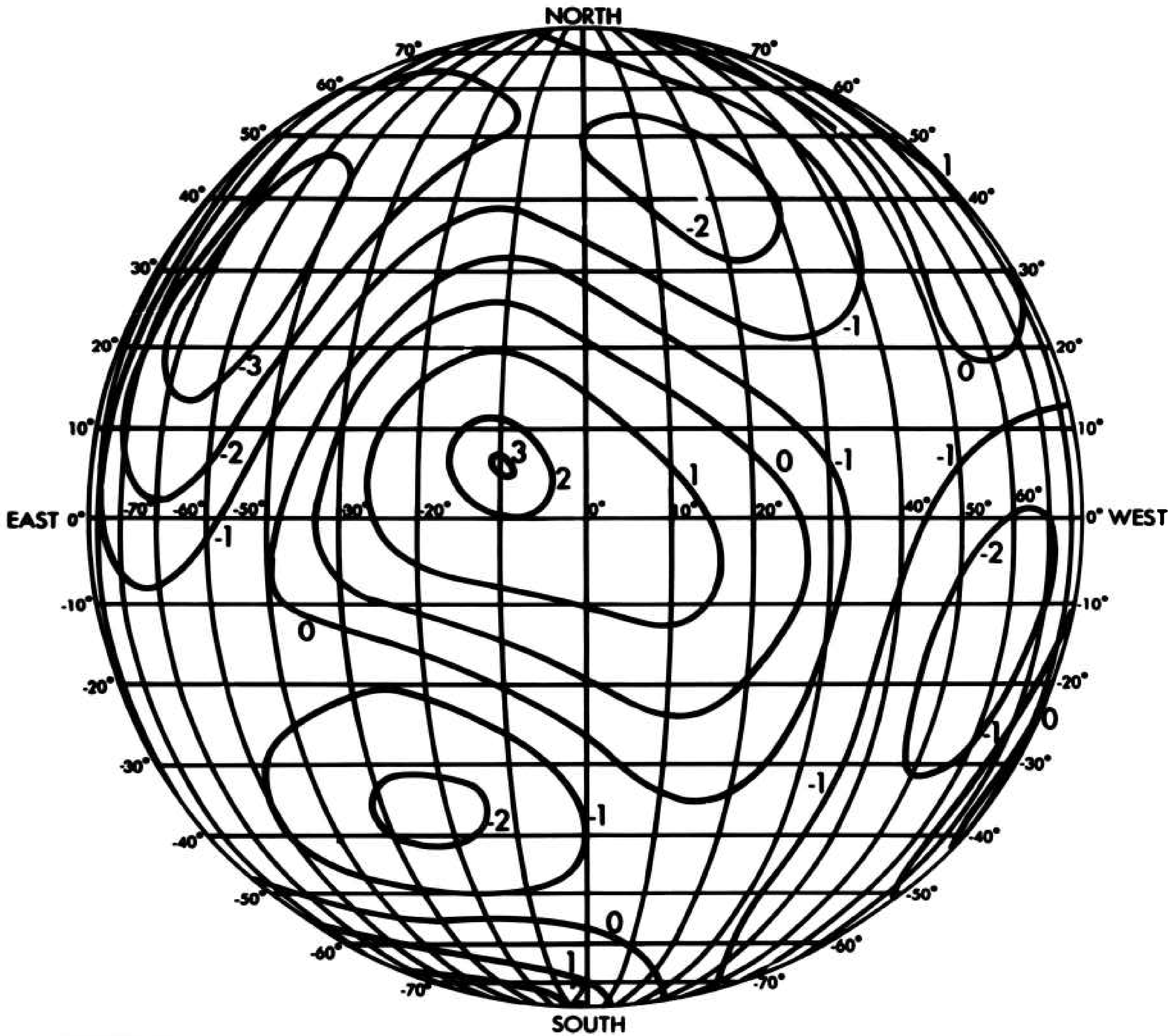

FIGURE II

CONTOUR MAP OF THE MOON BASED ON THE HARMONIC ANALYSIS OF SCHRUTKA'S (1958) ELEVATION DATA (SIX SURFACE HARMONICS ARE USED)

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and it becomes clear that the second assumption is worse than the first, because for both sets of data it led to larger deviations in the results from their expected values. A second conclusion is, of course, now inevitable. After this discussion it becomes obvious that the values of the harmonic coefficients depend on the assumption adopted for the far lunar side. For example, the values (8) were improved in the right direction by the adoption of the assumption that the lunar surface is symmetrical with respect to the plane $O\xi\eta$ although this improvement is very insufficient. The same change of assumption in the case of Schrutka's data led to an improvement for more substantial and resulted in values which can be considered completely satisfactory if one takes into consideration the estimated uncertainties due to errors in the data. But undoubtedly it may be possible to make an even better assumption that will approximate more the correct values (9) of the harmonics. Such an assumption can be e.g. a combination of the two assumptions already described.

Once the equation of surface is available and the values of J_{20} and J_{22} are found to be different from those expected, we put in their position the values given by Eq. (9). This is important because in this way we shall derive a contour map which will be compatible with the otherwise known values of f, β and γ. In more detail, this equation of surface and hence the contour map produced from it will have the same moments of inertia with the real Moon and also the same kinetic behavior in relation to librations. After this we proceed as follows: The equation of surface is,

$$r(\beta,\ell) = r_0\left[1 + \sum_{i=1}^{n}\sum_{j=0}^{i}(j_{ij} \cos j\ell + j'_{ij} \sin j\ell)T_{ij}(\beta)\right], \quad (10)$$

where r_0 in this case is the mean lunar radius adopted in the derivation of the elevation data plus the value J_{00}. We introduce the notation

$$f(\beta,\ell) = r_0 \sum_{i=1}^{n}\sum_{j=0}^{i}(j_{ij} \cos j\ell + j'_{ij} \sin j\ell)T_{ij}(\beta)$$

$$= \sum_{i=1}^{n}\sum_{j=0}^{i}(J_{ij} \cos j\ell + J'_{ij} \sin j\ell)T_{ij}(\beta) \ . \quad (11)$$

$f(\beta,\ell)$ gives in kilometers the elevation above or below the new mean level. An isolevel contour is any curve satisfying the equation

$$f(\beta,\ell) = k \ (\underline{\text{constant}}). \quad (12)$$

For $-\pi/2 \leq \beta \leq \pi/2$, $-\pi/2 \leq \ell \leq \pi/2$, there is a finite number of curves satisfying equation (12). There are a number of ways one can compute these curves numerically. We have used two such methods.

The first one was applied on an analogue computer at Cambridge Research Laboratories, Bedford, Massachusetts. The value of the $f(\beta,\ell)-k$ was first evaluated in the computer for a fixed value of β. Then the sign of this quantity was tested and the pen of a plotter following the variation of ℓ from $-\pi/2$ to $+\pi/2$ was printing a straight line on the paper whenever the sign was

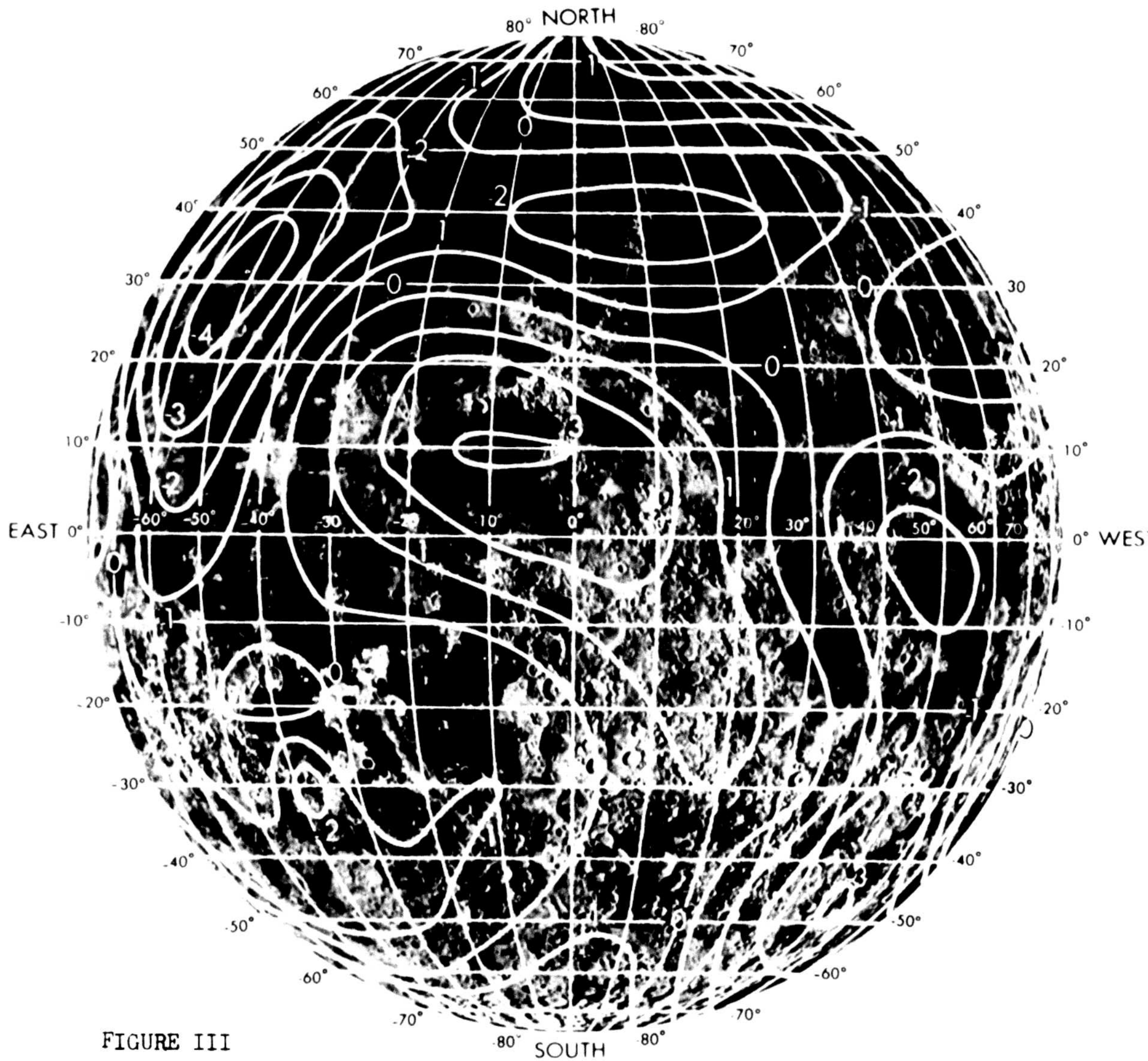

FIGURE III

CONTOUR MAP OF THE MOON BASED ON THE HARMONIC ANALYSIS OF SCHRUTKA'S (1958) ELEVATION DATA (EIGHT SURFACE HARMONICS ARE USED)

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negative and raised whenever it was positive. The same was repeated for all values of β from $-\pi/2$ to $+\pi/2$ always keeping k constant. In this way a pattern like the one shown in Figure I was derived. The terminating points of each horizontal line represent to an accuracy of one part in one thousand the zeros of the function $f(\beta,\ell)-k$ with β = constant, k = constant, in the interval $-\pi/2 \leq \ell \leq \pi/2$. If we connect the terminating points by a continuous line, we have an isolevel contour of elevation k kilometers. The contours in Figure I correspond, for example, to $k = 0$ km. and are based on Schrutka's (1958) data. If we now give different values to k, we shall obtain contours of different elevations which, if put in the same diagram, will represent a complete contour map.

This way of compiling a contour map seems to be very effective and for the time being sufficiently accurate, since the error in the derived contours is always less than a few meters and never more than ten.

The second way by which the contours of equal elevation can be traced numerically has already been described by this author (1963). A remark to be added here is that the values of J_{20} and J_{22} should be replaced by their correct ones in case the harmonic analysis did not produce the expected values. Also, it was realized from experience that instead of using the formulae

$$\frac{\partial f}{\partial \ell} = \frac{f(\ell+\delta\ell,\beta) - f(\ell,\beta)}{\delta\ell} , \qquad (13)$$

$$\frac{\partial f}{\partial \beta} = \frac{f(\ell,\beta+\delta\beta) - f(\ell,\beta)}{\delta\beta} , \qquad (14)$$

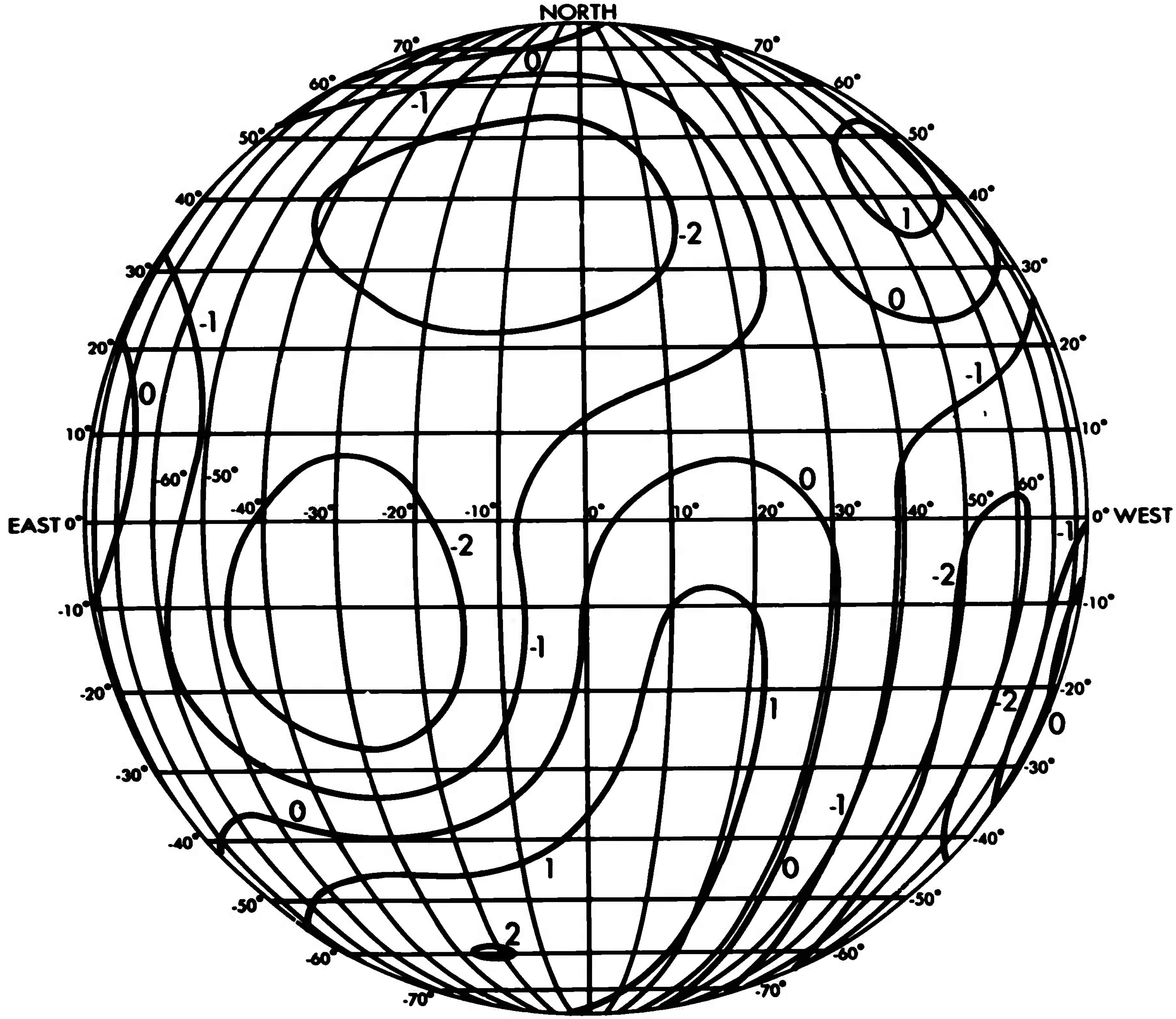

FIGURE IV

CONTOUR MAP OF THE MOON BASED ON THE HARMONIC ANALYSIS OF BALDWIN'S (1963) ELEVATION DATA (SIX SURFACE HARMONICS ARE USED)

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in computing the derivatives of $f(\beta,\ell)$ it is better to use the formulae,

$$\frac{\partial f}{\partial \ell} = \sum_{i=1}^{n} \sum_{j=0}^{i} j(-J_{ij} \sin j\ell + J'_{ij} \cos j\ell) T_{ij}(\beta), \tag{15}$$

$$\frac{\partial f}{\partial \beta} = \sum_{i=1}^{n} \sum_{j=0}^{i} (J_{ij} \cos j\ell + J'_{ij} \sin j\ell) \frac{d}{d\beta} T_{ij}(\beta)$$

$$= \sum_{i=1}^{n} \sum_{j=0}^{i} (J_{ij} \cos j\ell + J'_{ij} \sin j\ell) \left[T_{i,j+1}(\beta) - j\frac{\sin \beta}{\cos \beta} T_{ij}(\beta) \right], \tag{16}$$

which are exact.

The evaluation of the r.h.s. of formulae (15) and (16) is also easier from the programming point of view because it does not imply recomputation of the same functions for three different pairs (β,ℓ) as in the case of formulae (13) and (14), but only for one pair. The term $T_{i,j+1}(\beta)$ appearing in formula (13) does not produce additional difficulty because the expressions of $T_{ij}(\beta)$ are already computed up to the term $T_{nn}(\beta)$ and the term $T_{n,n+1}(\beta)$ is equal to zero.

Finally, the step of advancement along the same contour suggested in an earlier publication (Goudas, 1963, Eq. (66)) was found to be in certain cases too large or too small and it had to be varied according to circumstances.

III. A CONTOUR MAP FROM SCHRUTKA'S DATA

Both techniques described in the previous section were applied in order to reproduce the contour maps derived by Schrutka and Hopmann, and Baldwin, using the data employed by the above authors. No effort was made to make the results from the harmonic analysis of each set of data compatible with the most recent values of the mechanical ellipticity and other two relations among the moments of inertia of the Moon.

First, the data by Schrutka (1958) were treated putting in Equation (11) $n = 6$. The contour maps obtained from both techniques were virtually identical and are presented in Figure II. The same work was then repeated for $n = 8$. The contours obtained in this case were superimposed on a Mosaic photograph of the Moon kindly provided by Mr. Robert Carder of A.C.I.C. (U.S.A.F.), and the result is given in Figure III.

According to this map the deepest part of the near lunar side is the northeastern portion of Oceanus Procellarum, which is about 4 km. below the 1738.0 km. mean sphere. Almost all of Oceanus Procellarum is below level and so is Mare Serenitatis and Imbrium. The last two seas are not more than 2 km. below level. The area of Mare Nubium is also below level but not deeper than 1 km. Mare Huinorum is about at level. On the western hemisphere the deepest part is between Rheita Valley and crater Biela with elevation -3 km. Mare Foecunditatis is at elevation -2 km. and Mare Tranquilitatis and Serenitatis between 0 and -2 km.. The region of highest elevation is, according to this contour map, at Sinus Aestuum, which is about 3 km. above level. The whole part of the peninsula

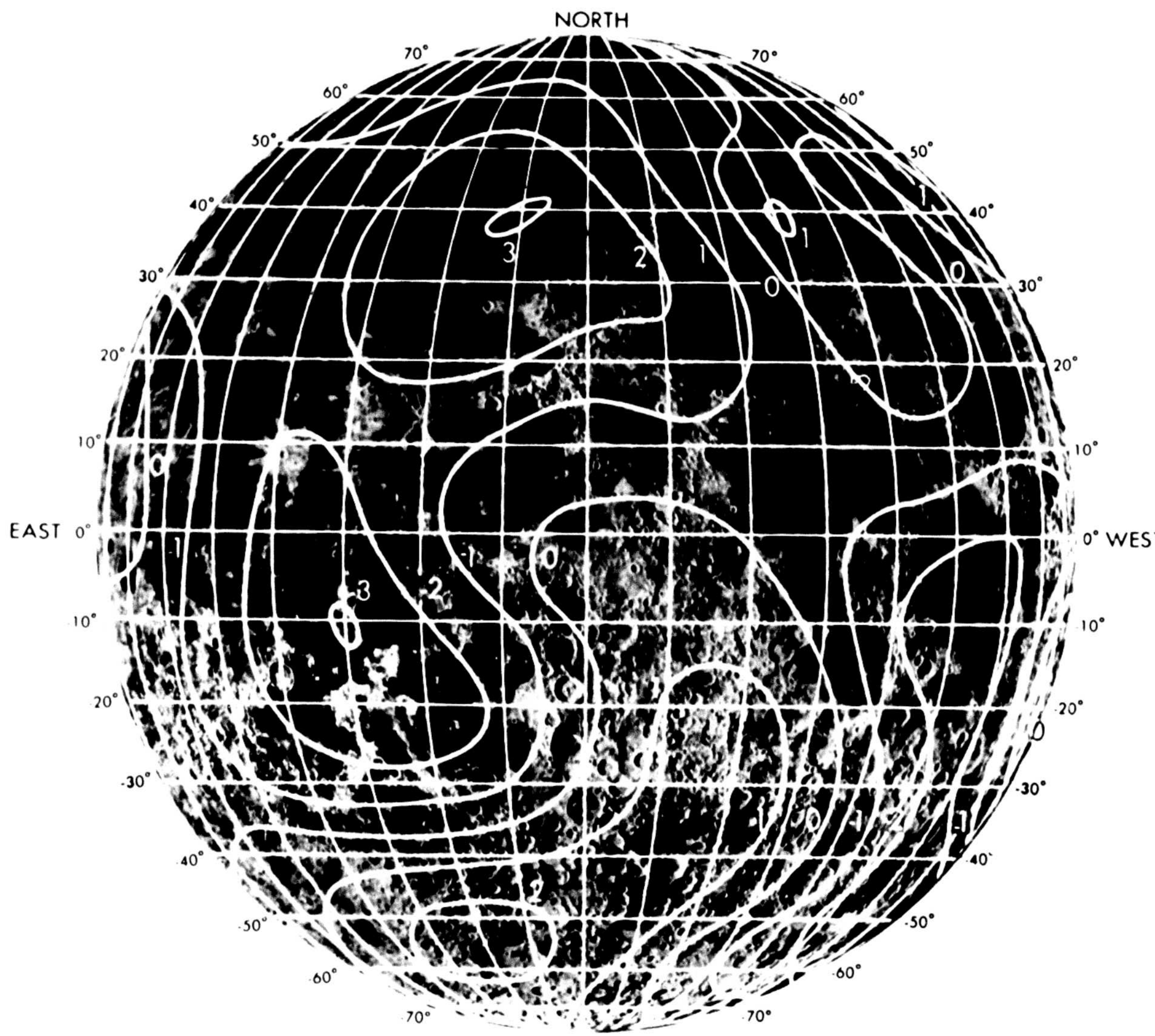

FIGURE V

CONTOUR MAP OF THE MOON BASED ON THE HARMONIC ANALYSIS OF BALWIN'S (1963) ELEVATION DATA (EIGHT SURFACE HARMONICS ARE USED)

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of Terrae extending from the south pole to about 30° positive latitude as well as some portions of the neighboring Maria are above level. The area between the south pole and crater Lilius is higher than 1 km. and so is a small area near the north pole and crater Meton. The last two high elevation regions together with the central bulge constitute the basis for the existence of a fourth zonal harmonic $J_{40}\, T_{40}\,(\beta)$ in the equation of surface and $K_{40}\left(\frac{r_0}{R}\right)^4 T_{40}(\beta)$ in the gravity field of the Moon, where $K_{40} = J_{40}/3r_0$. A study of the variation of altitudes along the equator shows again the reasons why fourth order tesseral harmonics should also exist.

IV. A CONTOUR MAP FROM BALDWIN'S DATA

The same procedure was followed in the case of Baldwin's data. In Figure IV we give the contour map obtained by taking $n = 6$. In Figure V we give the contour lines obtained in the same way and by taking $n = 8$ superimposed on another copy of the same Mosaic photograph used for Figure III. The transition from $n = 6$ to $n = 8$ does not seem to have any substantial effect on the pattern of the contours other than changing their elevation. This is due to the faster eonvergence of the coefficients of the harmonics with increasing n in the present case rather than in the case of Schrutka's data.

The contour map in Figure V seems to be a smoothed-out version of the map compiled by Baldwin where the smoothing came as a result of the harmonic analysis. The description of Baldwin's map with only small changes can therefore describe sufficiently well this map also. Inclusion of more terms in the harmonic expansion would result in a more detailed map but the author is convinced that a

map of absolute contours cannot be reliable to minute details on the basis of accuracies accessible today in elevation measurements. From this point of view we can appreciate more the map presented by Schrutka and Hopmann.

V. A BALANCED CONTOUR MAP

The equation of surface of the Moon obtained from an analysis of Schrutka's data (1958) was shown (Goudas, 1964) to be in agreement with the observed values of f, β and γ and as a consequence it is accepted as best representing the first eight harmonics of the lunar figure at present. The values of f, β and γ depend, however, on the two coefficients of the second zonal and sectorial harmonics and to a second order on the rest of the harmonic coefficients. As a result the compatibility of the data by Schrutka and the values of f, β and γ rests upon a comparison between the two known coefficients with their values, respectively, as given by the harmonic analysis. The rest of the coefficients obtained from the same analysis cannot be qualified since no previous knowledge about their size is available and we accept them as the best until improved values from the data provided by the Army Map Service and Schrutka-Rechtenstamm will be obtained.

The contour map derived by applying the technique described in a previous section and employing the harmonics from Schrutka's data, is shown in Figure VI. The difference between this map and the one given in Figure III where the second zonal and sectorial harmonics do not have their correct coefficients does not seem to be extremely large.

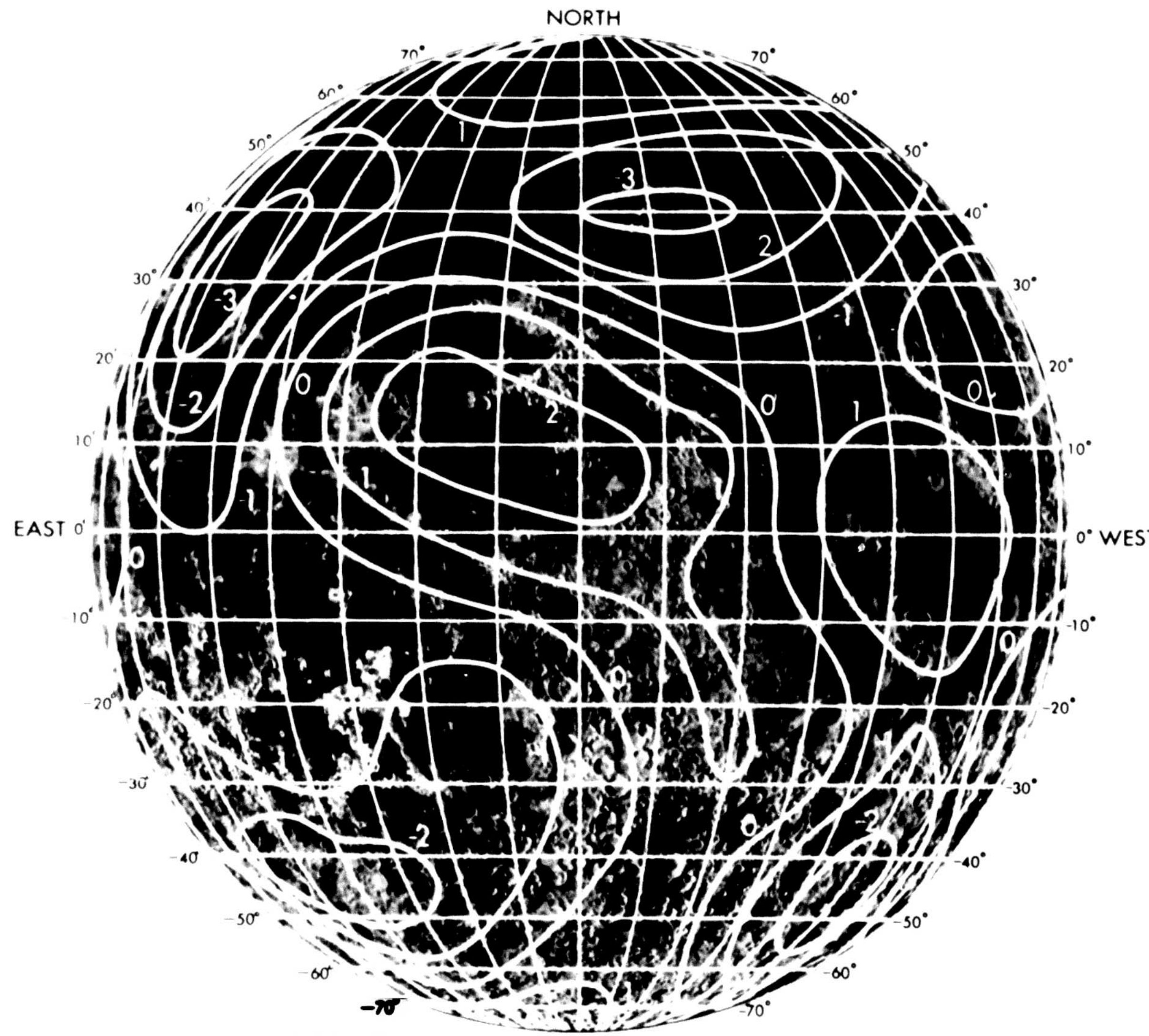

FIGURE VI

CONTOUR MAP OF THE MOON BASED ON THE 'MOST PROBABLE' SURFACE HARMONICS.

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The main merit of this contour map is that it represents a Moon with exactly the same values of f, β and γ as these are determined from librations and so gives a dynamically balanced body unlike any contour map hitherto published.

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APPENDIX

TABLES OF INTERPOLATED ELEVATIONS

$\ell = -1.570796$

β	h
-1.5708	1.0540
-1.3963	1.8470
-1.2217	1.8336
-1.0472	1.1520
-0.8727	0.0975
-0.6981	-0.9849
-0.5236	-1.5195
-0.3491	-0.8924
-0.1745	0.7650
0.0000	2.2635
0.1745	2.4346
0.3491	1.5284
0.5236	0.9079
0.6981	1.1330
0.8727	1.0437
1.0472	-0.6030
1.2217	-3.0402
1.3963	-4.0322
1.5708	-2.6191

$\ell = -1.396263$

β	h
-1.5708	1.0540
-1.3963	1.8296
-1.2217	1.7426
-1.0472	0.9169
-0.8727	-0.2237
-0.6981	-1.2230
-0.5236	-1.6026
-0.3491	-0.9742
-0.1745	0.4429
0.0000	1.6031
0.1745	1.5332
0.3491	0.5470
0.5236	-0.0230
0.6981	0.3744
0.8727	0.5739
1.0472	-0.7633
1.2217	-3.0159
1.3963	-3.9883
1.5708	-2.6191

$\ell = -1.221730$

β	h
-1.5708	1.0540
-1.3963	1.7809
-1.2217	1.5046
-1.0472	0.3174
-0.8727	-1.0115
-0.6981	-1.7401
-0.5236	-1.6674
-0.3491	-1.0058
-0.1745	-0.1763
0.0000	0.1878
0.1745	-0.3996
0.3491	-1.5407
0.5236	-2.0449
0.6981	-1.3484
0.8727	-0.5348
1.0472	-1.1347
1.2217	-2.9248
1.3963	-3.8582
1.5708	-2.6191

$\ell = -1.047197$

β	h
-1.5708	1.0540
-1.3963	1.7096
-1.2217	1.2096
-1.0472	-0.3857
-0.8727	-1.8635
-0.6981	-2.1478
-0.5236	-1.4336
-0.3491	-0.6855
-0.1745	-0.4857
0.0000	-0.8664
0.1745	-1.7820
0.3491	-2.9586
0.5236	-3.5384
0.6981	-2.8466
0.8727	-1.6152
1.0472	-1.4662
1.2217	-2.7261
1.3963	-3.6474
1.5708	-2.6191

ℓ = -0.872665

β	h
-1.5708	1.0540
-1.3963	1.6265
-1.2217	0.9588
-1.0472	-0.9077
-0.8727	-2.4109
-0.6981	-2.2433
-0.5236	-0.9808
-0.3491	-0.1688
-0.1745	-0.3561
0.0000	-0.9596
0.1745	-1.6417
0.3491	-2.5910
0.5236	-3.4455
0.6981	-3.2714
0.8727	-2.1337
1.0472	-1.5398
1.2217	-2.3885
1.3963	-3.3661
1.5708	-2.6191

ℓ = -0.698132

β	h
-1.5708	1.0540
-1.3963	1.5397
-1.2217	0.8314
-1.0472	-1.0636
-0.8727	-2.4934
-0.6981	-2.1343
-0.5236	-0.7547
-0.3491	-0.0211
-0.1745	-0.2155
0.0000	-0.3714
0.1745	-0.2812
0.3491	-0.8186
0.5236	-2.0738
0.6981	-2.7197
0.8727	-2.0361
1.0472	-1.3007
1.2217	-1.9177
1.3963	-3.0300
1.5708	-2.6191

ℓ = -0.523599

β	h
-1.5708	1.0540
-1.3963	1.4520
-1.2217	0.8454
-1.0472	-0.8237
-0.8727	-2.1447
-0.6981	-1.9965
-0.5236	-1.0427
-0.3491	-0.5380
-0.1745	-0.3863
0.0000	0.3319
0.1745	1.2598
0.3491	1.0484
0.5236	-0.6444
0.6981	-1.3914
0.8727	-1.6837
1.0472	-0.8746
1.2217	-1.3632
1.3963	-2.6596
1.5708	-2.6191

ℓ = -0.349066

β	h
-1.5708	1.0540
-1.3963	1.3595
-1.2217	0.9582
-1.0472	-0.3000
-0.8727	-1.4821
-0.6981	-1.7885
-0.5236	-1.5344
-0.3491	-1.2579
-0.1745	-0.5830
0.0000	0.9529
0.1745	2.3683
0.3491	1.9723
0.5236	-0.1604
0.6981	-1.8191
0.8727	-1.4946
1.0472	-0.4527
1.2217	-0.8008
1.3963	-2.2782
1.5708	-2.6191

ℓ = -0.174533

β	h
-1.5708	1.0540
-1.3963	1.2539
-1.2217	1.0827
-1.0472	0.3017
-0.8727	-0.6600
-0.6981	-1.2979
-0.5236	-1.5491
-0.3491	-1.3467
-0.1745	-0.2737
0.0000	1.5686
0.1745	2.7668
0.3491	1.8221
0.5236	-0.6898
0.6981	-2.3051
0.8727	-1.6273
1.0472	-0.1764
1.2217	-0.3050
1.3963	-1.9084
1.5708	-2.6191

ℓ = 0.174533

β	h
-1.5708	1.0540
-1.3963	0.9693
-1.2217	0.9984
-1.0472	0.8161
-0.8727	0.4491
-0.6981	0.2613
-0.5236	0.3832
-0.3491	0.6725
-0.1745	1.1318
0.0000	1.7101
0.1745	1.7027
0.3491	0.2590
0.5236	-2.0990
0.6981	-3.3421
0.8727	-2.2137
1.0472	-0.1342
1.2217	0.3171
1.3963	-1.2748
1.5708	-2.6191

ℓ = 0.000000

β	h
-1.5708	1.0540
-1.3963	1.1258
-1.2217	1.1206
-1.0472	0.7413
-0.8727	0.0887
-0.6981	-0.4897
-0.5236	-0.7569
-0.3491	-0.4968
-0.1745	0.5359
0.0000	1.9925
0.1745	2.5478
0.3491	1.0827
0.5236	-1.5682
0.6981	-2.9887
0.8727	-1.9450
1.0472	-0.0834
1.2217	0.0735
1.3963	-1.5696
1.5708	-2.6191

ℓ = 0.349066

β	h
-1.5708	1.0540
-1.3963	0.7854
-1.2217	0.6966
-1.0472	0.4449
-0.8727	0.1691
-0.6981	0.3891
-0.5236	0.9826
-0.3491	1.1947
-0.1745	0.8295
0.0000	0.5074
0.1745	0.3858
0.3491	-0.3640
0.5236	-2.0275
0.6981	-3.1769
0.8727	-2.3071
1.0472	-0.2660
1.2217	0.4373
1.3963	-1.0304
1.5708	-2.6191

$\ell = 0.523599$

β	h
-1.5708	1.0540
-1.3963	0.5822
-1.2217	0.2589
-1.0472	-0.2730
-0.8727	-0.7148
-0.6981	-0.3390
-0.5236	0.5535
-0.3491	0.6976
-0.1745	-0.2374
0.0000	-1.0358
0.1745	-0.8447
0.3491	-0.6200
0.5236	-1.5102
0.6981	-2.6639
0.8727	-2.2483
1.0472	-0.4238
1.2217	0.4660
1.3963	-0.8365
1.5708	-2.6191

$\ell = 0.698132$

β	h
-1.5708	1.0540
-1.3963	0.3743
-1.2217	-0.2246
-1.0472	-1.0763
-0.8727	-1.7882
-0.6981	-1.4966
-0.5236	-0.5072
-0.3491	-0.2672
-0.1745	-1.2198
0.0000	-1.9535
0.1745	-1.3435
0.3491	-0.4279
0.5236	-0.8368
0.6981	-2.0794
0.8727	-2.1129
1.0472	-0.5607
1.2217	0.4433
1.3963	-0.6887
1.5708	-2.6191

$\ell = 0.872665$

β	h
-1.5708	1.0540
-1.3963	0.1788
-1.2217	-0.6539
-1.0472	-1.6603
-0.8727	-2.4526
-0.6981	-2.2288
-0.5236	-1.2376
-0.3491	-0.8082
-0.1745	-1.4171
0.0000	-1.8303
0.1745	-1.0095
0.3491	0.0669
0.5236	-0.2400
0.6981	-1.5839
0.8727	-1.9328
1.0472	-0.6348
1.2217	0.4064
1.3963	-0.5800
1.5708	-2.6191

$\ell = 1.047198$

β	h
-1.5708	1.0540
-1.3963	0.0115
-1.2217	-0.9614
-1.0472	-1.8422
-0.8727	-2.3147
-0.6981	-1.8947
-0.5236	-0.9162
-0.3491	-0.4676
-0.1745	-0.8953
0.0000	-1.1671
0.1745	-0.4271
0.3491	0.4885
0.5236	0.1466
0.6981	-1.1935
0.8727	-1.6971
1.0472	-0.6232
1.2217	0.3802
1.3963	-0.5033
1.5708	-2.6191

$\ell = 1.396264$

β	h
-1.5708	1.0540
-1.3963	-0.1937
-1.2217	-1.2059
-1.0472	-1.3762
-0.8727	-0.5354
-0.6981	0.8484
-0.5236	1.7764
-0.3491	1.4449
-0.1745	0.1601
0.0000	-0.7739
0.1745	-0.4952
0.3491	0.3083
0.5236	0.2881
0.6981	-0.6612
0.8727	-1.1809
1.0472	-0.4505
1.2217	0.3753
1.3963	-0.4237
1.5708	-2.6191

$\ell = 1.221731$

β	h
-1.5708	1.0540
-1.3963	-0.1152
-1.2217	-1.1341
-1.0472	-1.6675
-0.8727	-1.4846
-0.6981	-0.5958
-0.5236	0.3629
-0.3491	0.4967
-0.1745	-0.2308
0.0000	-0.7432
0.1745	-0.2447
0.3491	0.5395
0.5236	0.2947
0.6981	-0.8815
0.8727	-1.4199
1.0472	-0.5429
1.2217	0.3721
1.3963	-0.4526
1.5708	-2.6191

$\ell = 1.570796$

β	h
-1.5708	1.0540
-1.3963	-0.2202
-1.2217	-1.2229
-1.0472	-1.2400
-0.8727	-0.1237
-0.6981	1.4710
-0.5236	2.3825
-0.3491	1.8282
-0.1745	0.2648
0.0000	-0.8755
0.1745	-0.6809
0.3491	0.1627
0.5236	0.2633
0.6981	-0.5792
0.8727	-1.0851
1.0472	-0.4103
1.2217	0.3781
1.3963	-0.4143
1.5708	-2.6191